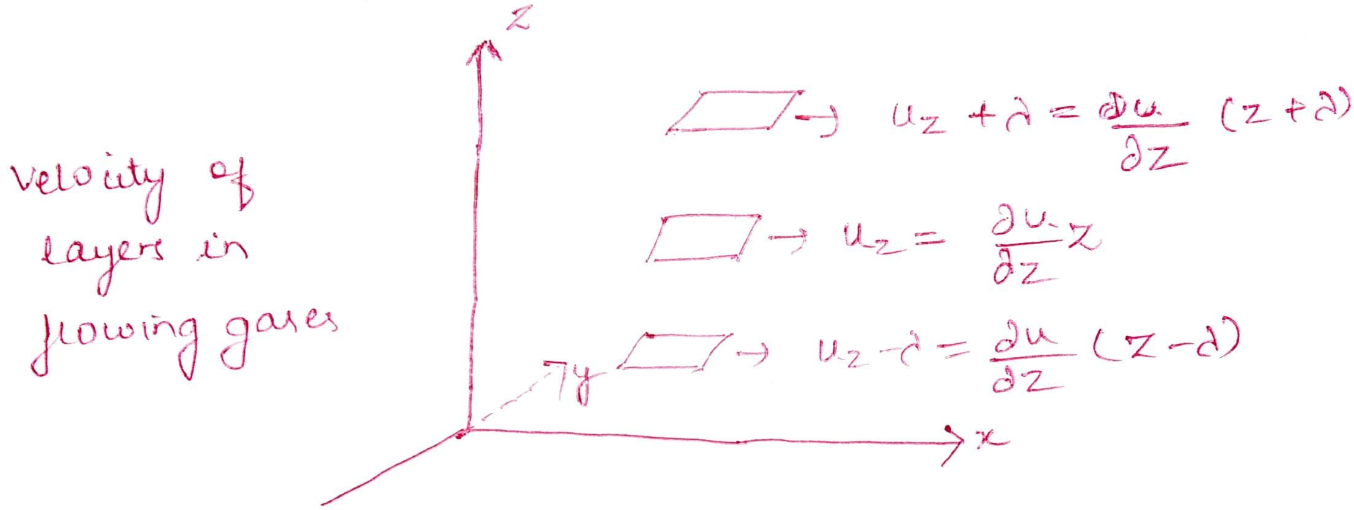
Viscosity of gases :-

The formula for the coefficient of viscosity of a gas can be derived in a way similar to that used for heat conduction.



Imagining two imaginary large parallel flat plates, one lying in the xy-plane, the other at a distance z above the xy-plane.

As we keep the lower plate stationary and pull the upper plate in +x direction with a velocity u. The viscosity of the gas exerts a drag on it. To keep the plate in uniform motion, there is a need of force to balance the viscous drag.

The upper plate moves with a velocity u and the viscous force will set lower plate in motion Therefore, a force is applied to the lower plate to keep it in the place.

Let us suppose that the gas between the plates is made up of horizontal layers. The layer next to the lower plate is immobile.

On moving upwards, each successive layer has a slightly larger component of velocity in x-direction. The topmost layer at a height Z having the velocity U. This type of flow, in which there is a regular gradation of velocity in passing from one layer to the next, is called <u>laminar flow.</u>

$$u_z = \frac{\partial u}{\partial z} z \quad —① $$

at $z = Z$ and $u = U$

$$\frac{\partial u}{\partial z} = \frac{U}{Z} \quad —② $$

On observing the layer at height ~~h~~ z, we can clearly see that the molecules enter from layers in the neighbour.

An extra momentum x is added to these layers from the upper layer and the lower one is deficient in the momentum. There is a net downward flow of momentum x through the layer.

For 1 m^2 of layer at height z, the flow rate can be calculated as:-

Number of molecules passing downwards
$$= \frac{1}{6} \rho \bar{c}$$

And no. of molecules downwards = no. of molecules going upwards.

Here ρ is the number density and $\bar{c}$ is mean speed of the molecules of the physical quantity.

The molecules that pass downwards through the layer at z carry momentum x appropriate to the layer in which they made their last collisions, the layer at height $z+d$. The momentum x can be given as

$$m u_{z+d} = m\left(\frac{\partial u}{\partial z}\right)(z+d) \quad \text{—} \ (3)$$

The downward momentum is given by

$$(mu)\downarrow = \frac{1}{6}\rho\bar{c}\left(\frac{\partial u}{\partial z}\right)(z+d) \quad \text{—} \ (4)$$

and upward momentum is given by

$$(mu)\uparrow = \frac{1}{6}\rho\bar{c}m\left(\frac{\partial u}{\partial z}\right)(z-d) \quad \text{—} \ (5)$$

Since the molecules coming up adjust their momentum in layer at $z-d$. The net downward flow of momentum is given by:-

$$(mu)\downarrow - (mu)\uparrow = \frac{1}{3}\rho\bar{c}md\left(\frac{\partial u}{\partial z}\right) \quad \text{—} \ (6)$$

Since this quantity is independent of z, it must also equal to net momentum x transferred in one second to $1m^2$ of the lower plate. given by

$$f(x) = \frac{1}{3}\rho\bar{c}md\left(\frac{du}{dz}\right) \quad \text{—} \ (7)$$

To hold this plate stationary, there is need of an equal and opposite force f_{-x} such that

$$\boxed{f_x + f_{-x} = 0} \quad \text{--- (8)}$$

Hence coefficient of viscosity η can be defined as,

$$f_{-x} = \eta \left(\frac{\partial u}{\partial z}\right) \quad \text{--- (9)}$$

The coefficient of viscosity can here be described as the force that must be applied to hold the lower plate stationary if velocity $\frac{\partial u}{\partial z}$ is unity and the plate has unit area.

Comparing eq. (6) and (7)

$$\eta = \frac{1}{3} \rho \bar{c} m \lambda \quad \text{--- (10)}$$

For a gas with mass density $= \rho_{mass}$

$$\rho_{mass} = \rho m \quad \text{--- (11)}$$

and Hence equation (10) becomes

$$\eta = \frac{1}{3} \rho_{mass} \bar{c} \lambda \quad \text{--- (12)}$$

As the flow of gas is non-equilibrium situation, the numerical factor does not hold much precise significance.

In case of elastic collision it becomes $\frac{1}{2}$.

The equation (10) can be simplified as

$$\eta = \frac{1}{3} \lambda \bar{c} \rho m = \frac{1}{3} \lambda \bar{c} \left(\frac{N}{V}\right) m = \frac{1}{3} \lambda \bar{c} \left(\frac{n N_A}{V}\right)\left(\frac{M}{N_A}\right)$$

$\left[\because n = \frac{N}{N_A} \text{ and } m = \frac{M}{N_A} \right]$. --- (13)

Therefore

$$\boxed{\eta = \frac{1}{3} \lambda \bar{c} M [A]} \quad \text{— (14)}$$

$[A]$ = the molar concentration of gas A
$= \frac{n}{V}$

- Viscosity of gas is independent of pressure (P)

At high P, more molecules are available for transport momentum but less far due to shorter mean free path.

- Viscosity increases with increase in temperature (T). since

$$\bar{c} \propto T^{1/2}$$

Also, as we can ascertain that high T, the molecules travel more quickly hence the flux of momentum is greater

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